

**Class X Session 2025-26**  
**Subject - Mathematics (Basic)**  
**Sample Question Paper - 04**

**Time Allowed: 3 hours**

**Maximum Marks: 80**

### General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1,1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take  $\pi = 22/7$  wherever required if not stated.
11. Use of calculators is not allowed.

## Section A

1. The HCF of the smallest 2-digit number and the smallest composite number is [1]  
a) 4  
b) 2  
c) 10  
d) 20
2. What is the largest number that divides each one of 1152 and 1664 exactly? [1]  
a) 64  
b) 128  
c) 32  
d) 256
3. The roots of the quadratic equation  $x^2 + 3x - 10 = 0$  are: [1]  
a) -5, 2  
b) 5, 2  
c) 5, -2  
d) -5, -2
4. If  $(-3, 2)$  is a solution of the linear equation  $5x + 3ky = 3$ , then the value of k is \_\_\_\_\_. [1]

a) 6

b) 3

c) 5

d) 2

5. The discriminant of the equation  $(2a + b)x = x^2 + 2ab$  is \_\_\_\_\_ [1]

a)  $(2a + b)^2$ b)  $(2a - b)^2$ c)  $(2a + b^2)$ d)  $(2a - b^2)$ 

6. If C(1, -1) is the mid-point of the line segment AB joining points A(4, x) and B(-2, 4), then value of x is: [1]

a) 6

b) -5

c) -6

d) 5

7. A vertical stick 1.8 m long casts a shadow 45 cm long on the ground. At the same time, what is the length of the shadow of a pole 6 m high? [1]

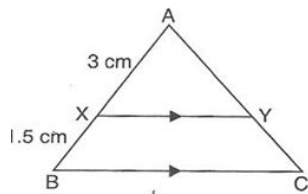
a) 2.4 m

b) 1.35 m

c) 1.5 m

d) 13.5 m

8. In the given figure  $XY \parallel BC$ . If  $AX = 3\text{ cm}$ ,  $XB = 1.5\text{ cm}$  and  $BC = 6\text{ cm}$ , then XY is equal to [1]



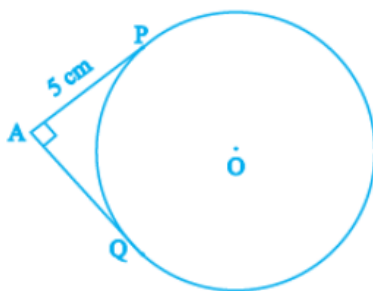
a) 6 cm.

b) 4.5 cm

c) 3 cm.

d) 4 cm.

9. The pair of tangents AP and AQ drawn from an external point to a circle with centre O are perpendicular to each other and length of each tangent is 5 cm. The radius of the circle is [1]



a) 10 cm

b) 5 cm

c) 7.5 cm

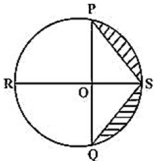
d) 2.5 cm

10. If  $5 \tan A = 3$ , then the value of  $\cot A$  is: [1]

a)  $\frac{4}{5}$ b)  $\frac{5}{3}$ c)  $\frac{3}{4}$ d)  $\frac{3}{5}$ 

11. The string of a kite in air is 50 m long and it makes an angle of  $60^\circ$  with the horizontal. Assuming the string to be straight, the height of the kite from the ground is: [1]

a)  $50\sqrt{3}\text{ m}$ b)  $25\sqrt{3}\text{ m}$ c)  $\frac{100}{\sqrt{3}}\text{ m}$ d)  $\frac{50}{\sqrt{3}}\text{ m}$

12. If  $\frac{x}{3} = 2 \sin A$ ,  $\frac{y}{3} = 2 \cos A$ , then the value of  $x^2 + y^2$  is: [1]  
 a) 6 b) 9  
 c) 36 d) 18
13. The hour hand of a clock is 6 cm long. The area swept by it between 11.20 am and 11.55 am is [1]  
 a)  $11 \text{ cm}^2$  b)  $2.75 \text{ cm}^2$   
 c)  $10 \text{ cm}^2$  d)  $5.5 \text{ cm}^2$
14. In the given figure PQ and RS are the perpendicular diameters of the circle whose centre is O and radius = 14 cm. the area of the shaded region is [1]
- 
- a)  $28 \text{ cm}^2$  b)  $35 \text{ cm}^2$   
 c)  $60 \text{ cm}^2$  d)  $112 \text{ cm}^2$
15. A bag contains cards numbered from 1 to 25. A card is drawn at random from the bag. The probability that the number on this card is divisible by both 2 and 3 is [1]  
 a)  $\frac{3}{25}$  b)  $\frac{1}{5}$   
 c)  $\frac{4}{25}$  d)  $\frac{2}{25}$
16. The median of first 8 prime numbers is [1]  
 a) 11 b) 13  
 c) 9 d) 7
17. The radii of the base of a cylinder and a cone are in the ratio 3 : 4. If they have their heights in the ratio 2 : 3, the ratio between their volumes is [1]  
 a) 8 : 9 b) 3 : 4  
 c) 9 : 8 d) 4 : 3
18. If the difference of mode and median of a data is 24, then the difference of median and mean of the same data is: [1]  
 a) 34 b) 12  
 c) 24 d) 8
19. **Assertion (A):** Distance between (5, 12) and origin is 13 units. [1]  
**Reason (R):**  $D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$   
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.  
 c) A is true but R is false. d) A is false but R is true.
20. **Assertion (A):** 3 is a rational number. [1]  
**Reason (R):** The square roots of all positive integers are irrationals.  
 a) Both A and R are true and R is the correct b) Both A and R are true but R is not the

explanation of A.

correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

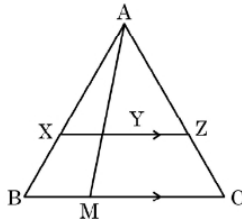
### Section B

21. Is the pair of linear equation consistent/inconsistent? If consistent, obtain the solution graphically:  $x - y = 8$ ;  $3x - 3y = 16$  [2]

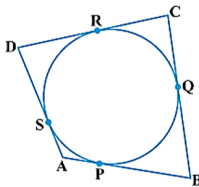
22. In  $\triangle ABC$ , D and E are the points on the sides AB and AC respectively such that  $DE \parallel BC$ . If  $AD = 6x - 7$ ,  $DB = 4x - 3$ ,  $AE = 3x - 3$  and  $EC = 2x - 1$ , find the value of x. [2]

OR

In the given figure, XZ is parallel to BC.  $AZ = 3$  cm,  $ZC = 2$  cm,  $BM = 3$  cm and  $MC = 5$  cm. Find the length of XY.



23. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that  $AB + CD = AD + BC$  [2]

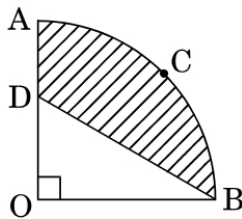


24. Prove that :  $\frac{(\sin^4 \theta + \cos^4 \theta)}{1 - 2 \sin^2 \theta \cos^2 \theta} = 1$ . [2]

25. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: [2]
- i. minor segment
  - ii. major sector.

OR

In Figure, OACB is a quadrant of a circle with centre O and radius 7 cm. If  $OD = 3$  cm, then find the area of the shaded region.



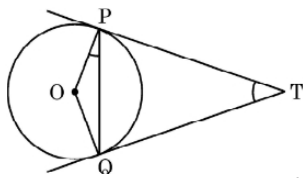
### Section C

26. Prove that  $3 + 2\sqrt{5}$  is irrational. [3]
27. If the points P, Q(x, 7), R, S(6, y) in this order divide the line segment joining A(2, p) and B(7, 10) in 5 equal parts, find x, y and p. [3]
28. If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes  $\frac{1}{2}$  if we only add 1 to the denominator. What is the fraction? Solve the pair of the linear equation obtained by the elimination method. [3]

OR

Two years ago father was five times as old as his son. Two years later, his age will be 8 years more than three times the age of the son. Find the present ages of father and son.

29. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that  $\angle PTQ = 2 \angle OPQ$ . [3]



30. If  $\tan \theta = \frac{12}{13}$ , evaluate  $\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$ . [3]

OR

Prove that  $\left( \frac{1+\tan^2 A}{1+\cot^2 A} \right) = \left( \frac{1-\tan A}{1-\cot A} \right)^2 = \tan^2 A$

31. Two different dice are thrown together. Find the probability that the numbers obtained [3]
- have a sum less than 7
  - have a product less than 16
  - is a doublet of odd numbers.

#### Section D

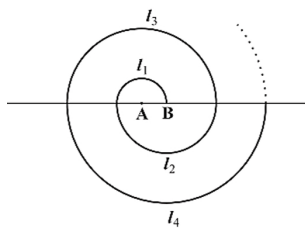
32. A train travels at a certain average speed for a distance of 360 km. It would have taken 48 minutes less to travel [5]  
the same distance if its speed was 5 km/hour more. Find the original speed of the train.

OR

Find the value of  $k$  for which the quadratic equation  $(k+1)x^2 - 2(3k+1)x + (8k+1) = 0$  has real and equal roots.

33. From the top of a vertical tower, the angles of depression of two cars in the same straight line with the base of [5]  
the tower, at an instant are found to be  $45^\circ$  and  $60^\circ$ . If the cars are 100 m apart and are on the same side of the tower, find the height of the tower.
34. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of [5]  
radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in Figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take  $\pi = \frac{22}{7}$ )

[Hint: Length of successive semicircles is  $l_1, l_2, l_3, l_4, \dots$  with centres at A, B, A, B, ... respectively.]



OR

The ratio of the 11<sup>th</sup> term to 17<sup>th</sup> term of an A.P. is 3: 4. Find the ratio of 5<sup>th</sup> term to 21<sup>st</sup> term of the same A.P. Also, find the ratio of the sum of first 5 terms to that of first 21 terms.

35. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying [5]  
number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50-52	53-55	56-58	59-61	62-64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

#### Section E

36. Read the following text carefully and answer the questions that follow: [4]

A wooden toy is shown in the picture. This is a cuboidal wooden block of dimensions  $14 \text{ cm} \times 17 \text{ cm} \times 4 \text{ cm}$ . On its top there are seven cylindrical hollows for bees to fit in. Each cylindrical hollow is of height 3 cm and

radius 2 cm.



- Find the volume of wood carved out to make one cylindrical hollow. (1)
- Find the lateral surface area of the cuboid to paint it with green colour. (1)
- Find the volume of wood in the remaining cuboid after carving out seven cylindrical hollows. (2)

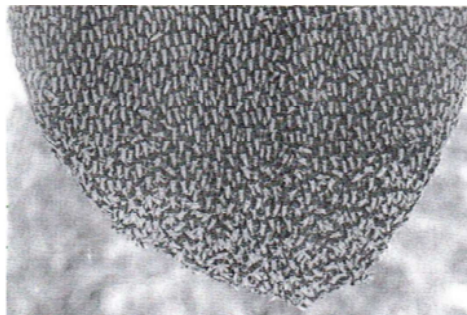
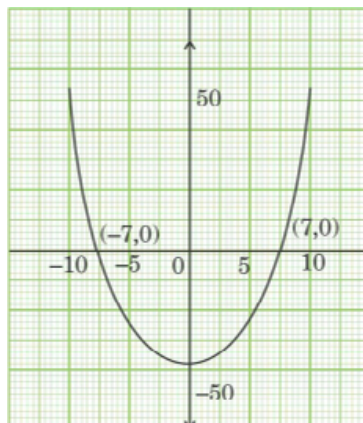
**OR**

Find the surface area of the top surface of the cuboid to be painted yellow. (2)

37. **Read the following text carefully and answer the questions that follow:**

[4]

While playing in a garden, Samaira saw a honeycomb and asked her mother what is that. Her mother replied that it's a honeycomb made by honey bees to store honey. Also, she told her that the shape of the honeycomb formed is a mathematical structure. The mathematical representation of the honeycomb is shown in the graph.



- How many zeroes are there for the polynomial represented by the graph given? (1)
- Write the zeroes of the polynomial. (1)
- If the zeroes of a polynomial  $x^2 + (a + 1)x + b$  are 2 and -3, then determine the values of a and b. (2)

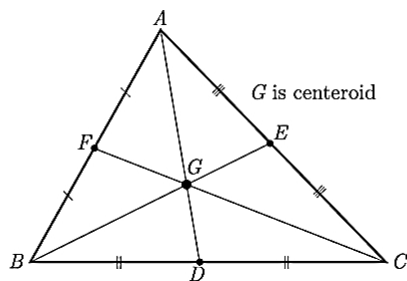
**OR**

If the square of difference of the zeroes of the polynomial  $x^2 + px + 45$  is 144, then find the value of p. (2)

38. **Read the following text carefully and answer the questions that follow:**

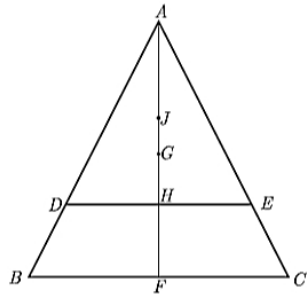
[4]

The centroid is the centre point of the object. It is also defined as the point of intersection of all the three medians. The median is a line that joins the midpoint of a side and the opposite vertex of the triangle. The centroid of the triangle separates the median in the ratio of 2 : 1. It can be found by taking the average of x-coordinate points and y-coordinate points of all the vertices of the triangle. See the figure given below



Here D, E and F are mid points of sides BC, AC and AB in same order. G is centroid, the centroid divides the median in the ratio 2 : 1 with the larger part towards the vertex. Thus  $AG : GD = 2 : 1$

On the basis of above information read the question below. If G is Centroid of  $\triangle ABC$  with height h and J is Centroid of  $\triangle ADE$ . Line DE parallel to BC, cuts the  $\triangle ABC$  at a height  $\frac{h}{4}$  from BC.  $HF = \frac{h}{4}$



- i. What is the length of AH? (1)
- ii. What is the distance of point A from point G? (1)
- iii. What is the distance of point A from point J? (2)

**OR**

What is the distance GJ? (2)

# Solution

## Section A

1.

**(b) 2**

**Explanation:**

Smallest two digit number is 10 and smallest composite number is 4 .

Clearly, 2 is the greatest factor of 4 and 10, so their H.C.F. is 2.

2.

**(b) 128**

**Explanation:**

Largest number that divides each one of 1152 and 1664 = HCF (1152, 1664)

We know,  $1152 = 2^7 \times 3^2$

$1164 = 2^7 \times 13$

$\therefore \text{HCF} = 2^7 = 128$

3.

**(a) -5, 2**

**Explanation:**

$$p(x) = x^2 + 3x - 10$$

$$= x^2 + 5x - 2x - 10$$

$$= x(x + 5) - 2(x + 5)$$

$$= (x - 2)(x + 5)$$

$$p(x) = 0$$

$$x - 2 = 0 \text{ or } x + 5 = 0$$

$$x = 2 \text{ or } x = -5$$

4.

**(b) 3**

**Explanation:**

Since, (-3,2) is the solution of  $5x + 3/cy = 3$ . So (-3, 2) satisfies it.

$$\therefore 5 \times (-3) + 3$$

$$\Rightarrow -15 + 6k = 3 \Rightarrow k = \frac{18}{6} = 3$$

5.

**(b)  $(2a - b)^2$**

**Explanation:**

$$(2a + b)x = x^2 + 2ab$$

$$x^2 - (2a + b)x + 2ab = 0$$

$$D = b^2 - 4ac$$

$$D = [-(2a + b)]^2 - 4 \times 1 \times 2ab$$

$$D = 4a^2 + b^2 + 4ab - 8ab$$

$$D = 4a^2 + b^2 - 4ab$$

$$D = (2a - b)^2$$

6.

**(c) -6**



**Explanation:**

A(4, x)                      C(1, 1)                      B(-2, 4)

Coordinate of C

$$C\left(\frac{4-2}{2}, \frac{x+4}{2}\right)$$

On comparing y coordinates.

$$-1 = \frac{x+4}{2}$$

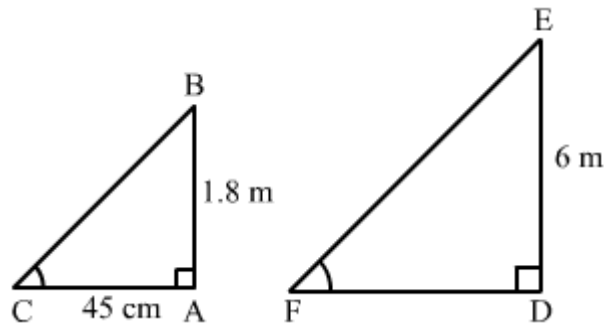
$$-2 = x + 4$$

$$x = -6$$

7.

(c) 1.5 m

**Explanation:**



Let AB and AC be the vertical stick and its shadow, respectively.

According to the question:

$$AB = 1.8 \text{ m}$$

$$AC = 45 \text{ cm} = 0.45 \text{ m}$$

Again, let DE and DF be the pole and its shadow, respectively.

According to the question:

$$DE = 6 \text{ m}$$

$$DF = ?$$

Now, in right-angled triangles ABC and DEF, we have:

$$\angle BAC = \angle EDF = 90^\circ$$

$$\angle ACB = \angle DFE \text{ (Angular elevation of the Sun at the same time)}$$

Therefore, by AA similarity theorem,

we get:  $\triangle ABC \sim \triangle DEF$

$$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF} \Rightarrow \frac{1.8}{0.45} = \frac{6}{DF} \Rightarrow DF = \frac{6 \times 0.45}{1.8} = 1.5 \text{ m}$$

8.

(d) 4 cm.

**Explanation:**

Since  $XY \parallel BC$ , then using Thales theorem,

$$\Rightarrow \frac{AX}{AB} = \frac{XY}{BC}$$

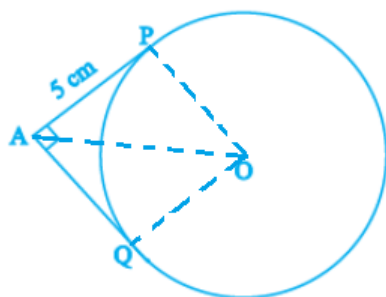
$$\Rightarrow \frac{3}{4.5} = \frac{XY}{6}$$

$$\Rightarrow XY = 4 \text{ cm}$$

9.

(b) 5 cm

**Explanation:**



$$AP = AQ = 5 \text{ cm}$$

(tangents from external point are equal)

Radius makes right angle with tangent

$\triangle APO \cong \triangle AQO$  (by R.H.S.)

As  $\angle PAQ = 90^\circ$ , So  $\angle PAO = 45^\circ$

In  $\triangle APO$

$$\tan 45^\circ = \frac{OP}{AP} = \frac{OP}{5}$$

$$\Rightarrow OP = 5 \text{ cm}$$

Hence, the radius of circle = 5 cm

10.

$$(b) \frac{5}{3}$$

**Explanation:**

$$5 \tan A = 3$$

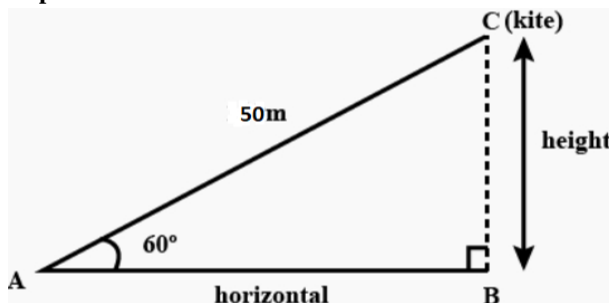
$$\tan A = \frac{3}{5}$$

$$\cot A = \frac{1}{\tan A} = \frac{5}{3}$$

11.

$$(b) 25\sqrt{3} \text{ m}$$

**Explanation:**



$$\sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin 60^\circ = \frac{BC}{AC} = \frac{h}{50}$$

$$\frac{\sqrt{3}}{2} = \frac{h}{50} \quad (\because \sin 60^\circ = \frac{\sqrt{3}}{2})$$

$$h = 25\sqrt{3} \text{ m}$$

12.

$$(c) 36$$

**Explanation:**

$$\frac{x}{3} = 2 \sin A, \frac{y}{3} = 2 \cos A$$

$$x = 6 \sin A, y = 6 \cos A$$

$$x^2 + y^2 = (6 \sin A)^2 + (6 \cos A)^2$$

$$= 36 \sin^2 A + 36 \cos^2 A$$

$$= 36 (\sin^2 A + \cos^2 A)$$

$$= 36(1)$$

$$= 36$$

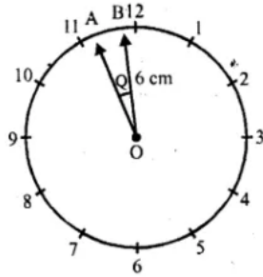
13.

(d)  $5.5 \text{ cm}^2$

**Explanation:**

Length of hour hand of a clock ( $r$ ) = 6 cm

Time 11.20 am to 11.55 am = 35 minute =  $\frac{35}{60} \text{ h}$



$\therefore$  In 1 hour the hour hand rotates  $30^\circ$ .

Thus, central angle of the sector =  $30 \times \frac{35}{60} = 17.5^\circ$

$\therefore$  Area of the sector swept by the hour hand =  $\frac{17.5}{360} \times \frac{22}{7} \times 6 \times 6 \text{ cm}^2$   
 $= \frac{2.5 \times 22}{10} \text{ cm}^2 = 5.5 \text{ cm}^2$

14.

(d)  $112 \text{ cm}^2$

**Explanation:**

For Triangle POS,

PO = OS = 14cm

Now Get PS by pythagoras theorem.

Again, Required area =  $\frac{1}{4}$  Area of Circle - Area POS

15.

(c)  $\frac{4}{25}$

**Explanation:**

Total number of outcomes = 25

The number which is divisible by both 2 and 3 are 6, 12, 18, 24

Number of favourable outcomes = 4

Probability of number which is divisible by both 2 and 3 =  $\frac{4}{25}$

16.

(c) 9

**Explanation:**

First 8 prime numbers are follows:

2, 3, 5, 7, 11, 13, 17, 19

N = 8 (even)

$$\begin{aligned} \therefore \text{Median} &= \frac{\left(\frac{8}{2}\right)^{\text{th}} \text{ value} + \left(\frac{8}{2} + 1\right)^{\text{th}} \text{ value}}{2} \\ &= \frac{4^{\text{th}} \text{ value} + 5^{\text{th}} \text{ value}}{2} \\ &= \frac{7 + 11}{2} \\ &= \frac{18}{2} \\ &= 9 \end{aligned}$$

17.

(c) 9 : 8

**Explanation:**

Let the radii of the base of the cylinder and cone be  $3r$  and  $4r$  and their heights be  $2h$  and  $3h$ , respectively.

$$\begin{aligned}\text{Then, ratio of their volumes} &= \frac{\pi(3r)^2 \times (2h)}{\frac{1}{3}\pi(4r)^2 \times (3h)} \\ &= \frac{9r^2 \times 2 \times 3}{16r^2 \times 3} \\ &= \frac{9}{8} \\ &= 9 : 8\end{aligned}$$

18.

**(b) 12**

**Explanation:**

Given,

$$\text{mode} - \text{median} = 24$$

$$\text{median} - \text{mean} = ?$$

we know that,

$$\text{mode} = 3 \text{ median} - 2 \text{ mean}$$

$$\text{mode} = \text{median} + 2 \text{ median} - 2 \text{ mean}$$

$$\text{mode} - \text{median} = 2 \text{ median} - 2 \text{ mean}$$

$$24 = 2 (\text{median} - \text{mean})$$

$$\text{median} - \text{mean} = \frac{24}{2} = 12$$

19. **(a) Both A and R are true and R is the correct explanation of A.**

**Explanation:**

$$\begin{aligned}\text{Distance of point } (5, 12) \text{ from } 8 \text{ origin is given, } d &= \sqrt{(5 - 0)^2 + (12 - 0)^2} \\ &= \sqrt{25 + 144} = \sqrt{169} = 13\end{aligned}$$

20.

**(c) A is true but R is false.**

**Explanation:**

Here, reason is not true.

$$\sqrt{9} = \pm 3, \text{ which is not an irrational number.}$$

A is true but R is false.

## Section B

$$21. x - y = 8 \dots\dots\dots(1)$$

$$3x - 3y = 16 \dots\dots\dots(2)$$

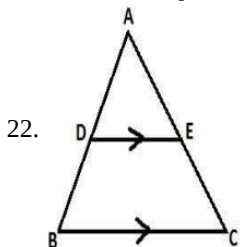
$$\text{Here, } a_1 = 1, b_1 = -1, c_1 = -8$$

$$a_2 = 3, b_2 = -3, c_2 = -16$$

$$\text{We see that } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the lines represented by the equations(1) and (2) are parallel.

Therefore, equations (1) and (2) have no solution, i.e., the given pair of linear equation is inconsistent.



22.

Given: In  $\triangle ABC$ ,  $DE \parallel BC$ . Also  $AD = 6x - 7$ ,  $DB = 4x - 3$ ,  $AE = 3x - 3$  and  $EC = 2x - 1$

By basic proportionality theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{6x-7}{4x-3} = \frac{3x-3}{2x-1}$$

$$\Rightarrow (6x - 7)(2x - 1) = (3x - 3)(4x - 3)$$

$$\Rightarrow 12x^2 - 6x - 14x + 7 = 12x^2 - 9x - 12x + 9$$

$$\Rightarrow -20x + 7 = -21x + 9$$

$$\Rightarrow -20x + 21x = 9 - 7$$

$$\Rightarrow x = 2$$

OR

Given that,

In the figure the triangle ABC

$XZ \parallel BC$  and the length of  $AZ = 3$  cm,  $ZC = 2$  cm,  $BM = 3$  cm and  $MC = 5$  cm.

From  $\triangle ABC$  and  $\triangle AXZ$

$$\angle AXZ = \angle ABC \text{ [by corresponding angles]}$$

$$\angle AZX = \angle ACB \text{ [by corresponding angles]}$$

By basic proportionality theorem  $\triangle ABC$  and  $\triangle AXZ$  are similar.

So,

$$\frac{YZ}{MC} = \frac{AZ}{ZC}$$

$$\frac{YZ}{5} = \frac{3}{2}$$

$$YZ = \frac{15}{2}$$

Then,

$$\frac{XZ}{BC} = \frac{AZ}{ZC}$$

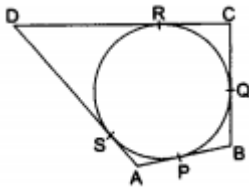
$$\frac{XY + YZ}{BM + MC} = \frac{AZ}{ZC}$$

$$\frac{XY + \frac{15}{2}}{3 + 5} = \frac{3}{2}$$

$$XY + \frac{15}{2} = \frac{24}{2}$$

$$XY = \frac{9}{2} = 4.5 \text{ cm}$$

23.



We know that the lengths of tangents drawn from an exterior point to a circle are equal.

$$AP = AS, \dots \text{(i) [tangents from A]}$$

$$BP = BQ, \dots \text{(ii) [tangents from B]}$$

$$CR = CQ, \dots \text{(iii) [tangents from C]}$$

$$DR = DS, \dots \text{(iv) [tangents from D]}$$

$$AB + CD = (AP + BP) + (CR + DR)$$

$$= (AS + BQ) + (CQ + DS) \text{ [using (i), (ii), (iii), (iv)]}$$

$$= (AS + DS) + (BQ + CQ)$$

$$= AD + BC.$$

$$\text{Hence, } AB + CD = AD + BC.$$

24. We have,

$$\begin{aligned} & \frac{(\sin^4 \theta + \cos^4 \theta)}{1 - 2 \sin^2 \theta \cos^2 \theta} \\ &= \frac{(\sin^2 \theta)^2 + (\cos^2 \theta)^2}{1 - 2 \sin^2 \theta \cos^2 \theta} \\ &= \frac{(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta}{1 - 2 \sin^2 \theta \cos^2 \theta} [(a + b)^2 = a^2 + b^2 + 2ab] \\ &= \frac{1 - 2 \sin^2 \theta \cos^2 \theta}{1 - 2 \sin^2 \theta \cos^2 \theta} [\sin^2 \theta + \cos^2 \theta = 1] \\ &= 1 \text{ Hence proved} \end{aligned}$$

25. i.  $r = 10$  cm,  $\theta = 90^\circ$

$$\text{Area of minor sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{90}{360} \times 3.14 \times 10 \times 10 = 78.5 \text{ cm}^2$$

$$\text{Area of } \triangle OAB = \frac{OA \times OB}{2}$$

$$= \frac{10 \times 10}{2} = 50 \text{ cm}^2$$

$\therefore$  Area of the minor segment

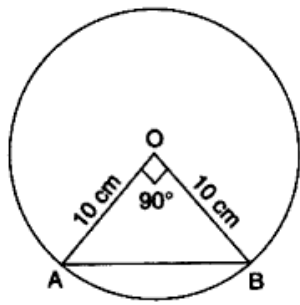
$$= \text{Area of minor sector} - \text{Area of } \triangle OAB$$

$$= 78.5\text{cm}^2 - 50\text{cm}^2 = 28.5\text{cm}^2$$

$$\text{ii. Area of major sector} = \pi r^2 - \text{area of minor sector}$$

$$= 3.14 \times 10 \times 10 - 78.5$$

$$= 314 - 78.5 = 235.5\text{cm}^2$$



OR

$$\text{Area of quadrant} = \frac{1}{4}\pi(7)^2 = \frac{49}{4}\pi \text{ cm}^2$$

$$\text{Area of triangle} = \frac{1}{2} \times 7 \times 3 = \frac{21}{2} \text{ cm}^2$$

$$\text{Area of shaded region} = \frac{49}{4}\pi - \frac{21}{2}$$

$$= \frac{7}{2} \left( \frac{7}{2}\pi - 3 \right) \text{ cm}^2 \text{ or } 28 \text{ cm}^2$$

### Section C

26. Let us assume, to the contrary, that is  $3 + 2\sqrt{5}$  rational.

That is, we can find coprime integers a and b ( $b \neq 0$ ) such that

$$3 + 2\sqrt{5} = \frac{a}{b} \text{ Therefore, } \frac{a}{b} - 3 = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2}$$

Since a and b are integers,

We get  $\frac{a}{2b} - \frac{3}{2}$  is rational, also so  $\sqrt{5}$  is rational.

But this contradicts the fact that  $\sqrt{5}$  is irrational.

This contradiction arose because of our incorrect assumption that  $3 + 2\sqrt{5}$  is rational.

So, we conclude that  $3 + 2\sqrt{5}$  is irrational.

27.

Consider QB.

The point S divides QB in the ratio 2:1

$$6 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$\Rightarrow 6 = \frac{2(7) + 1(x)}{2+1}$$

$$\Rightarrow 6 = \frac{x+14}{3}$$

$$\Rightarrow 18 = x + 14$$

$$\Rightarrow x = 4$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\Rightarrow y = \frac{2(10) + 1(7)}{2+1}$$

$$\Rightarrow y = \frac{20+7}{3}$$

$$\Rightarrow y = 9$$

Consider AS.

The point Q divides AS in the ratio 1:1.

$$7 = \frac{y_1 + y_2}{2}$$

$$\Rightarrow 7 = \frac{p+9}{2}$$

$$\Rightarrow 14 = p + 9$$

$$\Rightarrow p = 5$$

So,  $x = 4$ ,  $y = 9$ ,  $p = 5$ .

28. Let the fraction be  $\frac{x}{y}$

Then, according to the question,

$$\frac{x+1}{y-1} = 1 \dots\dots(1)$$

$$\frac{x}{y+1} = \frac{1}{2} \dots\dots(2)$$

$$\Rightarrow x + 1 = y - 1 \dots\dots\dots(3)$$

$$2x = y + 1 \dots\dots\dots(4)$$

$$\Rightarrow x - y = -2 \dots\dots\dots(5)$$

$$2x - y = 1 \dots\dots\dots(^)$$

Substituting equation (5) from equation (6), we get  $x = 3$

Substituting this value of  $x$  in equation (5), we get

$$3 - y = -2$$

$$\Rightarrow y = 3 + 2$$

$$\Rightarrow y = 5$$

Hence, the required fraction is  $\frac{3}{5}$

Verification: Substituting the value of  $x = 3$  and  $y = 5$ ,

we find that both the equations(1) and (2) are satisfied as shown below:

$$\frac{x+1}{y-1} = \frac{3+1}{5-1} = \frac{4}{4} = 1$$

$$\frac{x}{y+1} = \frac{3}{5+1} = \frac{3}{6} = \frac{1}{2}$$

Hence, the solution is correct.

OR

Let the age of father is  $x$  years and that of son is  $y$  years.

Then by the given question,

Two years ago father was five times as old as his son

$$x - 2 = 5(y - 2)$$

$$\text{or, } x - 5y = -10 + 2$$

$$\text{or, } x - 5y = -8$$

$$\text{or, } x = 5y - 8$$

Two years later, his age will be 8 years more than three times the age of the son

$$x + 2 = 3(y + 2) + 8$$

$$\text{or, } x - 3y = 6 + 8 - 2$$

$$\text{or, } 5y - 8 - 3y = 12$$

$$\text{or, } 2y = 12 + 8$$

$$\text{or, } y = \frac{20}{2}$$

$$\text{or, } y = 10$$

$$\text{then } x = 5y - 8 = 50 - 8 = 42$$

Then, the age of father is 42 yrs. and the age of son is 10 yrs.

29. Given : A circle with centre  $O$  and an external point  $T$  and two tangents  $TP$  and  $TQ$  to the circle, where  $P, Q$  are the points of contact.

To Prove:  $\angle PTQ = 2\angle OPQ$

Proof: Let  $\angle PTQ = \theta$

Since  $TP, TQ$  are tangents drawn from point  $T$  to the circle.

$$TP = TQ$$

$\therefore$   $TPQ$  is an isoscles triangle

$$\therefore \angle TPQ = \angle TQP = \frac{1}{2} (180^\circ - \theta) = 90^\circ - \frac{\theta}{2}$$

Since,  $TP$  is a tangent to the circle at point of contact  $P$

$$\therefore \angle OPT = 90^\circ$$

$$\therefore \angle OPQ = \angle OPT - \angle TPQ = 90^\circ - (90^\circ - \frac{1}{2} \theta) = \frac{\theta}{2} = \frac{1}{2} \angle PTQ$$

Thus,  $\angle PTQ = 2\angle OPQ$

30. We have,  $\tan \theta = \frac{12}{13}$

$$\text{Now, } \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{\frac{2 \sin \theta \cos \theta}{\cos^2 \theta}}{\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}} \quad [\text{dividing numerator and denominator by } \cos^2 \theta]$$

$$= \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \times \frac{12}{13}}{1 - \left(\frac{12}{13}\right)^2} = \frac{\frac{24}{13}}{1 - \frac{144}{169}} = \frac{\frac{24}{13}}{\frac{25}{169}} = \frac{24}{13} \times \frac{169}{25} = \frac{312}{25}$$

$$\text{Hence, } \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{312}{25}$$

OR

First, we will show that,  $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \tan^2 A$

$$\text{LHS} = \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \tan^2 A}{1 + \frac{1}{\tan^2 A}} = \frac{1 + \tan^2 A}{\frac{1 + \tan^2 A}{\tan^2 A}}$$

$$= (1 + \tan^2 A) \times \frac{\tan^2 A}{1 + \tan^2 A}$$

$$= \tan^2 A = \text{RHS} \dots (i)$$

Now, we will show that,  $\left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$

$$\text{LHS} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \left(\frac{1 - \tan A}{1 - \frac{1}{\tan A}}\right)^2 = \left(\frac{1 - \tan A}{\frac{\tan A - 1}{\tan A}}\right)^2$$

$$= \left[(1 - \tan A) \times \left(\frac{\tan A}{-(1 - \tan A)}\right)\right]^2$$

$$= (\tan A)^2 = \tan^2 A = \text{RHS} \dots (ii)$$

Hence, from (i) and (ii),

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$$

Hence proved.

31. When 2 dice are rolled,

The possible outcomes are :

(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)

(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)

(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)

(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)

(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)

(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)

∴ Total number of outcomes = 36

i. Let A be the event of getting the numbers whose sum is less than 7.

Number of favourable outcomes = 15

Favourable outcomes are (1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2) and (5,1).

$$\therefore P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{15}{36} = \frac{5}{12}$$

ii. Let B be the event of getting the numbers whose product is less than 16.

Number of favourable outcomes = 25

Favourable outcomes are (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(6,1) and (6,2).

$$\therefore P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{25}{36}$$

iii. Let C be the event of getting the numbers which are doublets of odd numbers.

Number of favourable outcomes = 3

Favourable outcomes are (1,1),(3,3) and (5,5).

$$\therefore P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{36} = \frac{1}{12}$$

## Section D

32. Given that a train travelling at a uniform speed for 360 km

Let the original speed of the train be x km/hr

$$\text{Time taken} = \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x}$$

$$\text{Time taken at increased speed} = \frac{360}{x+5} \text{ hours.}$$

According to the question

$$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$$

$$360 \left[ \frac{1}{x} - \frac{1}{x+5} \right] = \frac{4}{5}$$



$$\text{or, } \frac{360(x+5-x)}{x^2+5x} = \frac{4}{5}$$

$$\text{or, } \frac{1800}{x^2+5x} = \frac{4}{5}$$

$$\Rightarrow x^2 + 5x - 2250 = 0$$

$$\Rightarrow x^2 + (50 - 45)x - 2250 = 0$$

$$\Rightarrow x^2 + 50x - 45x - 2250 = 0$$

$$\Rightarrow (x + 50)(x - 45) = 0$$

Either  $x = -50$  or  $x = 45$

As speed cannot be negative

$\therefore$  Original speed of train = 45 km/hr.

OR

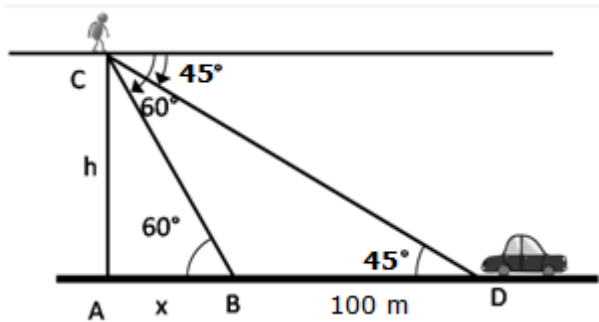
For real and equal roots.

$$[-2(3k + 1)]^2 - 4(k + 1)(8k + 1) = 0$$

$$\Rightarrow k^2 - 3k = 0$$

$$\therefore k = 0, k = 3$$

33.



Let AC be the tower of height =  $h$  m.

In right  $\triangle DAC$ ,

$$\cot 45^\circ = \frac{AD}{AC}$$

$$\Rightarrow 1 = \frac{x+100}{h}$$

$$\Rightarrow x + 100 = h$$

$$\Rightarrow x = h - 100 \dots(i)$$

In right  $\triangle BAC$ ,

$$\cot 60^\circ = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{h}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \dots(ii),$$

From (i) and (ii),

$$h - 100 = \frac{h}{\sqrt{3}}$$

$$\Rightarrow h - \frac{h}{\sqrt{3}} = 100$$

$$\Rightarrow h \left(1 - \frac{1}{\sqrt{3}}\right) = 100$$

$$\Rightarrow h \left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 100$$

$$\Rightarrow h = \frac{100\sqrt{3}}{\sqrt{3}-1}$$

$$h = \frac{100\sqrt{3}}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$\Rightarrow h = \frac{300+100\sqrt{3}}{2}$$

$$= 100 \left(\frac{3+\sqrt{3}}{2}\right)$$

$$= 50(3 + \sqrt{3})$$

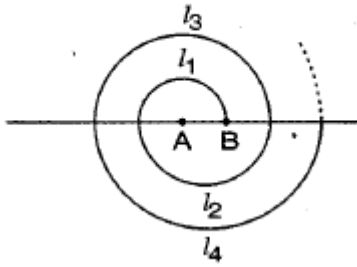
$$= 50(3 + 1.73)$$

$$= 236.5m$$

Hence, the height of the tower is 236.5 m.

34. According to question we are given that a spiral is made up of successive semi-circles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ....as shown in Fig.

Let  $l_1, l_2, l_3, l_4, \dots, l_{13}$  be the lengths (circumferences) of semi-circles of radii  $r_1 = 0.5$  cm,  $r_2 = 1.0$  cm,  $r_3 = 1.5$  cm,  $r_4 = 2.0$  cm,  $r_5 = 2.5$  cm, ... respectively.



Now, Semi-perimeter of circle =  $\pi \cdot r$

Therefore,

$$l_1 = \pi r_1 = \pi \times 0.5 = \frac{\pi}{2} \text{ cm}$$

$$l_2 = \pi r_2 = \pi \times 1 = 2 \left( \frac{\pi}{2} \right) \text{ cm}$$

$$l_3 = \pi r_3 = \pi \times \frac{3}{2} = 3 \left( \frac{\pi}{2} \right) \text{ cm}$$

$$l_4 = \pi r_4 = \pi \times 2 = 4 \left( \frac{\pi}{2} \right) \text{ cm}$$

and

$$l_{13} = \pi r_{13} = \pi \times \frac{13}{2} \text{ cm} = 13 \left( \frac{\pi}{2} \right) \text{ cm}$$

Therefore total length of the spiral =  $l_1 + l_2 + l_3 + \dots + l_{13}$

$$= \left\{ \frac{\pi}{2} + 2 \left( \frac{\pi}{2} \right) + 3 \left( \frac{\pi}{2} \right) + \dots + 13 \left( \frac{\pi}{2} \right) \right\}$$

$$= \frac{\pi}{2} (1 + 2 + 3 + \dots + 13)$$

$$= \frac{\pi}{2} \times \frac{13}{2} (1 + 13) \quad \left[ \text{Using } S_n = \frac{n}{2} (a + 1) \right]$$

$$= \frac{\pi}{2} \times \frac{13}{2} \times 14 = \frac{1}{2} \times \frac{22}{7} \times 13 \times 7 = 143 \text{ cm}$$

which is required length of the spiral made up of thirteen consecutive semi-circles.

OR

$$\text{Given } \frac{a+10d}{a+16d} = \frac{3}{4}$$

$$\Rightarrow 4a + 40d = 3a + 48d$$

$$\Rightarrow a = 8d$$

$$\text{therefore } \frac{a_5}{a_{21}} = \frac{a+4d}{a+20d} = \frac{3}{7} \text{ using (i)}$$

$$a_5 : a_{21} = 3 : 7$$

$$\frac{s_5}{s_{21}} = \frac{\frac{5}{2}(2a+4d)}{\frac{21}{2}(2a+20d)} = \frac{5 \times 20d}{21 \times 36d} = \frac{25}{189}$$

$$\text{Therefore, } S_5 : S_{21} = 25 : 189$$

35. Since value of number of mangoes and number of boxes are large numerically. So we use step-deviation method

True Class Interval	No. of boxes( $f_i$ )	Class mark( $x_i$ )	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
49.5-52.5	15	51	-2	-30
52.5-55.5	110	54	-1	-110
55.5-58.5	135	57	0	0
58.5-61.5	115	60	1	115
61.5-64.5	25	63	2	50
	$\sum f_i = 400$			$\sum f_i u_i = 25$

Let assumed mean ( $a$ ) = 57,

$$h = 3,$$

$$\therefore \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{25}{400} = 0.0625 \text{ (approx.)}$$

Using formula, Mean ( $\bar{x}$ ) =  $a + h\bar{u}$

$$= 57 + 3(0.0625)$$

$$= 57 + 0.1875$$

$$= 57.1875$$

$$= 57.19 \text{ (approx)}$$

Therefore, the mean number of mangoes is 57.19

### Section E

36. i. Volume of wood carved out to make one hollow

$$= \frac{22}{7} \times 2 \times 2 \times 3 = \frac{264}{7} \text{ cm}^3 \text{ or } 37.7 \text{ cm}^3$$

ii. LSA of cuboid =  $2(14 \times 4 + 17 \times 4) = 248 \text{ cm}^2$ .

iii. Volume of 7 cylindrical hollows =  $264 \text{ cm}^3$ .

$$\text{Volume of original cuboid} = 14 \times 17 \times 4 = 952 \text{ cm}^3.$$

$$\therefore \text{Volume of remaining solid} = 952 - 264 = 688 \text{ cm}^3.$$

**OR**

$$\text{Area of top surface to be painted} = (l \times b) - 7 \times \pi r^2$$

$$= (14 \times 17) - \left(\frac{22}{7} \times 4 \times 7\right)$$

$$= 150 \text{ cm}^2$$

37. i. Two

ii. 7 and -7

iii.  $-(a + 1) = 2 + (-3) \Rightarrow a = 0$

$$b = 2 \times (-3) \Rightarrow b = -6$$

**OR**

Let  $\alpha$  and  $\beta$  be the zeroes of given polynomial

$$\text{Here, } \alpha + \beta = -p \text{ and } \alpha\beta = 45$$

$$(\alpha - \beta)^2 = 144$$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 144$$

$$\Rightarrow (-p)^2 - 4 \times 45 = 144$$

$$\Rightarrow p = \pm 18$$

38. i.  $\therefore AF = h$  (Given)

$$\therefore AF = AH + HF$$

$$h = AH + \frac{h}{4}$$

$$AH = h - \frac{h}{4}$$

$$AH = \frac{3h}{4}$$

ii.  $\therefore AF = h$  (Given)

$$\therefore AG = \frac{2}{3} AF$$

$\therefore$  centroid divide the median in 2 : 1

iii.  $AH = \frac{3h}{4}$

J is centroid of  $\triangle ADE$

$$AJ : JH = 2 : 1$$

let  $AJ = 2x$  and  $JH = x$

$$2x + x = \frac{3h}{4}$$

$$x = \frac{h}{4}$$

$$AJ = 2 \times \frac{h}{4} = \frac{h}{2}$$

$$AG = AJ + GJ$$

$$= \frac{h}{2} + \frac{h}{6}$$

$$= \frac{2h}{3}$$

$$\text{But } AJ = \frac{h}{2} \times \frac{2}{3}$$

$$AJ = \frac{3}{4} AG$$

**OR**

$$GJ = AG - AJ$$

$$= AG - \frac{3}{4} AG$$

$$GJ = \frac{1}{4} AG$$